Optimizing classification thresholds of status of transionospheric communication channel distributed according to rayleigh distibution law for decreased quadrocopter's positioning errors

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Abstract

Unmanned aerial vehicle (UAV), today, plays prominent role in many parts of human activity. One of serious problems exploitation UAV is accuracy of positioning. Satellite clocks, satellite orbits, ionosphere and troposphere, multipath, etc - all of this are sources of signal's fluctuation. But largest fluctuation of signal is created by ionosphera. One of distribution laws (Nakagami, normal distribution law, Rayleigh and Rice - most common situations) characterize Ionosphere's fluctuations. For solve this problem supposed to using of adapting control system, based on Flight Control Center. At working this system there exist probability of making type I and type II errors during salvation of identification problems. This work proposes an approach to implementing the threshold optimization for classifying the states of satellite communication systems, when disorting of signals described by Rayleigh distribution law.

1 Introduction

When receiving signals in satellite communication channels there frequently appears a fluctuation of the signal amplitude in time. The fluctuations usually take place as a random process with a qausiperiod from fractions of a second to dozens of minutes, and their main descriptive feature is fading depth. The fading depth is defined by the deviation of the instantaneous amplitude of the signal from the set level (as a rule, median one) and can reach dozens of decibels. The level of the fluctuating signal can be estimated with the statistical method. In

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general, the nonstationary fading process is usually divided into two stationary processes: with fluctuations of the medium field values and fast fluctuations around medium values. The first type of fluctuations refers to slow fading, whereas the second type refers to the fast one [Taa16].

The most common distribution of the signal amplitude at fast interference fading is close to Rayleigh distribution.

The energy loss in a satellite communication channel, when there is Rayleigh fading, turns out to be quite substantial [Pin19]. Therefore, it is necessary to provide the probability of the wrong data reception, in order to keep the quality of communication in case of a Rayleigh fading not less than 10-5.

One of the methods to keep the set probability of the wrong data receipt at the necessary level is to use an automated quality control and management system, which will assess the state of the transitionspheric communication channels in the mode close to real time. Based on the transitionspheric communication channels state assessment, it is possible to identify the required control action which will allow to keep the necessary probability of the wrong reception of a GPS/GLONASS signal, and thereby to provide the required accuracy of remotely piloted aircraft system positioning. Thus, it is necessary to solve the task of the transionspheric communication channels state identification at Rayleigh fading in order to develop the appropriate control actions leading to the required remotely piloted aircraft system positioning accuracy.

During the transionspheric communication channels state identification, errors of the first and second kind may appear, which may lead to an inappropriate control action or to an untimely reaction to the change in the transionspheric communication channels state, resulting in the deterioration of the remotely piloted aircraft system positioning accuracy.

2 Analysis

Let us analyze the factors that influence the remotely piloted aircraft system positioning accuracy. The main error types and the inaccuracies in the remotely piloted aircraft system positioning caused by them are given in Table 1 [Ste12].

Table 1: The main error types and the inaccuracies in the remotely piloted aircraft system positioning

Error type	Caused inaccuracy (m)
Ephemeris data	from 1 to 5
Satellite clock	Up to 3
Ionosphere influence	From 2 to 50
Troposphere effect	Around 1
Signal reflection	Around 1
Receiver measurements	Up to 0.5

It is obvious that ionosphere has the most influence on the remotely piloted aircraft system positioning accuracy. Satellite communication channels are largely influenced by ionosphere due to the electron density and its changes, which leads to high level Rayleigh fading in satellite radio-signals. The ratio signal/noise (S/N) experiences fading depending on the changes in electron density.

If the fading level in a communication channel is rather high, i.e. the number of square components is big, and none of them exceeds in its amplitude all the rest, then the amplitude signal distribution is defined by Rayleigh distribution, which is described by [Lam10]:

$$p_R(r) = \frac{r}{\sigma^2} exp(-\frac{r^2}{2\sigma^2})$$

where r is the received signal amplitude; 2σ is the average signal energy.

The most common error metric in satellite communication channels with character oriented messaging without antinoise coding is error probability per message bit. Modern requirements of the International Communication Union set limits for this parameter in the required in practice value not less, than 10^{-5} [Ion16]. The error probability is defined [Ion16]:

$$P_{err} = 0.5 \cdot \frac{k_1^2}{k_1^2 + h_{b0}^2} \cdot exp(-\frac{k^2 + h_{b0}^2}{k_1^2 + h_{b0}^2})$$

where h_{b0}^2 is a normalized ratio signal/noise at receiver input; k - is the ratio of effective voltage of the regular and diffuse signal components at the receiver input, where $k_1^2 = 1 + k2$.

When the regular component is absent at the receiver input (scattering in the communication channel), for a rather little error probability and high h_{b0} values, the probability of the wrong receipt equals [Ion16]:

$$P_{err} = 0.5 \cdot \frac{k_1^2}{h_{b0}^2} \cdot exp(-k^2)$$

With typical for practice values of $k = 57dB$ [Ion16], the losses in the Rayleigh channel are 7-14 dB higher than in channel with Rician fading, which is a considerable value. Antinoise coding, implemented in channels with permanent parameters, at slow fading provides an advantage of several dB [Ion16], which does not provide a compensation of the antinoise loss. An automatic control and management system is to carry out the controlled object condition identification, to work out the control action (based on the management goals and taking into account the current condition of the environment), to realize the control action in the automated mode close to real time. This will provide a timely reaction to the changes in the condition of transionospheric communication channels characterized by high uncertainty. When identifying the controlled object condition there appear errors of the first and the second kind. In statistical hypothesis testing a type I error is the rejection of a true null hypothesis (also known as a "false positive" finding or conclusion), while a type II error is the non-rejection of a false null hypothesis (also known as a "false negative" finding or conclusion). It is necessary to determine the optimal value of the classification threshold as the error data directly influence the object positioning accuracy. The article suggests a solution for the optimization of the thresholds of the first and second type error classification for an automated system of control and management of transionospheric communication channels, the condition of which is described by the Rayleigh distribution law.

3 Problem Statement

In general, the task aimed to identify objects could be reduced to the verification of numerous hypotheses $H_1, H_2, \ldots, H_i, \ldots, H_n$, where H_i is a hypothesis implying the objects belonging to Class A_i . Lets assume that the a priori distributions of these hypotheses probabilities are set. It is known whats the likelihood the object $P(H_i)$ can belong to class A_i (or how often the object of this class is appeared). Moreover, $\sum_{i=1}^{n} P(H_i) = 1$, as the object is to be pertained to a certain class. The conditional density of distribution is $p_i(x) = p(x_i/H_i)$.

Two hypotheses $H_1 = N$ and $H_2 = \bar{N}$ are used in the identification system under design at corresponding to them a priori probabilities of situation, emerging in the network as a normal $p_1 = p(H_1) = p(N)$ one and an abnormal $p_2 = p(H_2) = p(\bar{N})$. And also $p_1 + p_2 = 1$.

It is required to find a decision rule ensuring the top accuracy in the identification system. Using the Neyman-Pearson criterion we fix the probability of "false alarm" $P_{f.a.}$ at stable level C and claim the minimum of pass error P_{pass}^{min} of TN operating trouble. Then

$$P_{pass}^{min} = p_2[1 - \prod_{i=1}^{n} \bar{\beta}(x_{oi})] \qquad (1)$$

at restriction of $P_{f.a.} = p_1 \prod_{i=1}^{n} \alpha(x_{oi}) = C$, const, where $\alpha(x_0)$ are Type I errors; and $\beta(x_0)$ are Type II errors.

4 Solution

During operation of transionospheric communication channels the status of which could be described by a large number of parameters, performance monitoring of their working efficiency is to be carried out using stage-by-stage principle of classification.

At the first stage, the transionospheric communication channels status is checked by the summarized index and if the abnormality is detected, a stricter control is carried out at the second and next stages on which its real status is defined.

At each of the stages the control system makes Type I $\alpha(x_o i)$ and Type II $\beta(x_o i)$ errors. Type I and II errors ("false alarm" and "abnormal situation" errors), are defined as follows:

$$\alpha(x_{oi}) = \int_{x_{oi}}^{\infty} f(x_i/N)dx_i \tag{2}$$

$$\beta(x_{oi}) = \int_{-\infty}^{x_{oi}} f(x_i/\bar{N})dx_i \tag{3}$$

The activity of the monitoring error evaluation system can be described by a probability graph.

The transionospheric communication channels are characterized with the certain states of N and $\bar{N}$, under which we will mean normal and abnormal operation in the process of its functioning which (in the monitoring system) are respectively displayed into the normal A and abnormal $^{-}$state.

A priori probabilities of the normal and abnormal states in transionospheric communication channels are detected correspondingly by a priori probabilities of the p_1 and p_2 states. For all the stages, with abnormal situation omission, were obtained resultant errors:

$$P_{pass} = p_2\beta_1 + p_2\bar{\beta}_1\beta_2 + \cdots + p_2\bar{\beta}_1\bar{\beta}_2 \ldots \beta_n = p_2[1 - \prod_{i=1}^{n} \bar{\beta}(x_{oi})] \tag{4}$$

and "false alarm" errors:

$$P_{f.a} = p_1\alpha_1\alpha_2 \ldots \alpha_n = p_1 \prod_{i=1}^{n} \alpha(x_{oi}) \tag{5}$$

where:$\bar{\beta}_i = \bar{\beta}(x_{oi})$, $\alpha_i = \alpha(x_{oi})$, $p_1 = 1 - p_2$ a priori probability of a normal situation occurrence; p_2 a priori probability of its absence.

The problem of thresholding x_{oi} for each stage is currently central. Lets make tradeoffs of thresholds. Following the Neyman-Pearson criterion we will set the probability of false alarm at certain given C level. Then, for the entire network, we get

$$P_{f.a} = p_1 \prod_{i=1}^{n} \alpha(x_{oi}) = C \tag{6}$$

Having minimized the probability of abnormal situation omission we get

$$P_{pass}^{min} = \min_{x_{oi}} p_2[1 - \prod_{i=1}^{n} \bar{\beta}(x_{oi})] \tag{7}$$

As a result, the decision rule ensuring the highest level of accuracy for the identification system is the Neyman-Pearson criterion which is detected for the TN by expressions (6) and (7). The minimization problem of the function (7), where variables xoi are linked by functional dependence (6) is the constrained optimization problem. Lets form a functional of optimization

$$F = p_2[1 - \prod_{i=1}^{n} \bar{\beta}(x_{oi})] + \lambda p_1 \prod_{i=1}^{n} \alpha(x_{oi}) \tag{8}$$

where λ is an undetermined Lagrange multiplier.

Calculating the derivatives $\frac{dF}{dx_{oi}} = 0$ we get a system of n equations:

$$\begin{gathered} p_2 \frac{d\bar{\beta}(x_{o1})}{dx_{o1}} \bar{\beta}(x_{o2}) \ldots \bar{\beta}(x_{on}) - \lambda p_1 \frac{d\alpha(x_{o1})}{dx_{o1}} \alpha(x_{o2}) \ldots \alpha(x_{on}) = 0; \\ \ldots \\ p_2 \frac{d\bar{\beta}(x_{on})}{dx_{on}} \bar{\beta}(x_{o1}) \ldots \bar{\beta}(x_{o(n-1)}) + \lambda p_1 \frac{d\alpha(x_{on})}{dx_{on}} \alpha(x_{o1}) \ldots \alpha(x_{o(n-1)}) = 0; \end{gathered} \tag{9}$$

which, along with equation (6) allow to find the undetermined Lagrange multiplier λ and n of x_{oi} variables.

Since the system (9) contains n of x_{oi} variables and undetermined Lagrange multiplier λ, which must be defined, while the equation system (9) has n equations and $n+1$ of variables, then for finding its unambiguous solution one more equation should be added, in the capacity of which we take the tolerance probability equation for "false alarm" (6).

Equations (9) and (6) including (2) and (3) after differentiation on the lower and higher threshold take the form:

$$\frac{\int_{-\infty}^{x_{o2}} f(x_2/\bar{N})dx_2 \dots \int_{-\infty}^{x_{on}} f(x_n/\bar{N})dx_n}{\int_{x_{o2}}^{\infty} f(x_2/N)dx_2 \dots \int_{x_{on}}^{\infty} f(x_n/N)dx_n} = \lambda \frac{p_1 f(x_{o1}/N)}{p_2 f(x_{o1}/\bar{N})};$$

$$\dots \tag{10}$$

$$\frac{\int_{-\infty}^{x_{o1}} f(x_1/\bar{N})dx_1 \dots \int_{-\infty}^{x_{o(n-1)}} f(x_{n-1}/\bar{N})dx_{n-1}}{\int_{x_{o1}}^{\infty} f(x_1/N)dx_1 \dots \int_{x_{o(n-1)}}^{\infty} f(x_{n-1}/N)dx_{n-1}} = \lambda \frac{p_1 f(x_{on}/N)}{p_2 f(x_{on}/\bar{N})};$$

$$p_1 \prod_{i=1}^{n} \int_{x_{oi}}^{\infty} f(x_i/N)dx_i = C.$$

Under certain distribution laws and $f(x_i/\bar{N})$ this problem has a single value solution. In equations (10) the unknowns are optimal thresholds for classification $(x^*_{o1}, x^*_{o2}, \dots, x^*_{oi}, \dots, x^*_{on})$ at each of the stages, offering a minimum of the function (7), i.e. the minimal probability of abnormal situation omission. The solution clearly allows defining the probability of transionospheric communication channels operating trouble absence (probability of normal operation):

$$\bar{P}_{pass} = 1 - P^{min}_{pass} \tag{11}$$

At each of the following stages, the information about correct solution of $\bar{\beta}_i = 1 - \beta_i$ (information about normal transionospheric communication channels functioning) is analyzed. When the Type I or Type II errors are detected, their specification is carried out by means of the following stage procedure. Let us define the procedure of error detection by the feature x as the first stage of two-stage procedure, while for the second stage we will detect the error by the feature y. Generally, the identification problem solution is determined by Type I and Type II errors. Now lets write down the functions for distribution density of the feature x at transionospheric communication channels problem-free functioning $f(x/I = f_1(x))$ and at trouble functioning $f(x/\bar{I} = f_1(x))$. Then the errors of Type I and Type II of the detector (stage 1) are :

$$\alpha_o = \int_{x_o}^{\infty} f_1(x)dx \ \beta_o = \int_{-\infty}^{x_o} f_2(x)dx \tag{12}$$

Type II errors for the recognizer (Stage 2) are defined:

$$\alpha_p = \int_{y_o}^{\infty} f_1(y)dy \ \beta_p = \int_{-\infty}^{y_o} f_2(y)dy \tag{13}$$

Transionospheric communication channels has the following conditions: "1" - the system is out of order, the failure was not detected; "2" the system is operational, it was found as workable; "3" - failure was detected and recognized (abnormal situation presence); "4" the system is workable, false detection and recognition (false alarm); "5" system is out of order, failure was detected but not recognized (abnormal situation omission); "6" - the system is in order, a false detection and correct recognition.

In view of formulae (12) and (13), formulae (6) and (7) take the form:

$$P_{f.a.} = p_1 \int_{x_o}^{\infty} f_1(x)dx \int_{y_o}^{\infty} f_1(y)dy = C = const \tag{14}$$

$$P_{pass}^{min} = minp_2(1 - \int_{x_o}^{\infty} f_2(x)dx \int_{y_o}^{\infty} f_2(y)dy) \tag{15}$$

Since in the expression (14) the thresholds x_o and y_o are linked with one functional dependence $x_o = \phi(y_o)$, having differentiated (15) by y_o and set it to zero, we get:

$$\frac{dx_o}{dy_o} \cdot f_2(x_o) \cdot \int_{y_o}^{\infty} f_2(y)dy + f_2(y_o) \cdot \int_{x_o}^{\infty} f_2(x)dx = 0 \tag{16}$$

At free-hand laws of x and y features distribution, particularly, at normal law, there is no possibility to obtain an exact solution of the classification thresholds optimization problem. However, in certain cases, at Rayleigh distribution laws, in particular, the solution could be obtained in its final form.

Lets set densities of features x and y distribution probability in the form of Rayleigh distribution laws. The Type I and Type II errors are in the form of:

$$\alpha_o = \int_{x_o}^{\infty} f_1(x)dx;\ \beta_o = \int_{a}^{x_o} f_2(x)dx;\ \alpha_p = \int_{y_o}^{\infty} f_1(y)dy;\ \beta_p = \int_{b}^{y_o} f_2(y)dy;$$

$$f_1(x) = xe^{-\frac{x^2}{2}};\ f_2(x) = (x-a)e^{-\frac{(x-a)^2}{2}};\ f_1(y) = ye^{-\frac{y^2}{2}};\ f_2(y) = (y-b)e^{-\frac{(y-b)^2}{2}};$$

Equations (14) and (16) are transformed into expressions:

$$x_o^2 + y_o^2 = 2ln\frac{p_1}{C} \tag{17}$$

$$\frac{dx_o}{dy_o} \cdot (x_o - a) + (y_o - b) = 0 \tag{18}$$

Having differentiated (17) by $_o$ by substituting the result in (18), we obtain optimal classification thresholds:

$$x_o^* = \sqrt{\frac{2lnp_1/C}{1+(\frac{b}{a})^2}};\ y_o^* = \sqrt{\frac{2lnp_1/C}{1+(\frac{a}{b})^2}} \tag{19}$$

5 Example

Lets analyze increasing UAV positioning errors, using program u-center. Analized data was got in real experiment from UAV.

Fig. 1 demonstrate example of UAV positioning errors distribution, when navigation area is fluctuating.

Fig. 1.left demonstrating example of UAV positioning errors distribution for clear sky, i.e. normal state of navigation area. UAV positioning errors less than 0,9 m. That data was get with PDOP (Position dilution of precision) = 1, signal/noise ratio in measuring channels - not less than 45 dBm / Hz, the time delay of the signal is $1 \ldots 2$ ns.

Fig 1.right demonstrating example of UAV positioning errors distribution caused by ionosphere fluctuation. UAV positioning errors more than 7 m. That data was get with PDOP = $1 \ldots 2$, signal/noise ratio in measuring channels $33 \ldots 28$ dBm / Hz, the time delay of the signal is $2 \ldots 5$ ns.

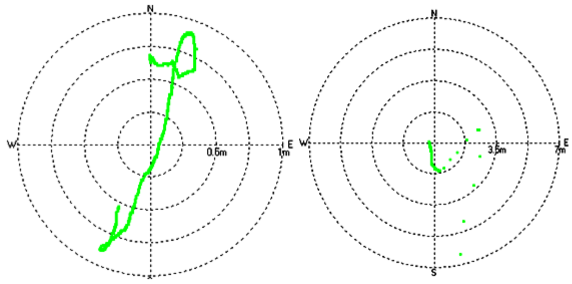

Figure 1: Positioning error increase of UAV into ionospheric fluctuations area.

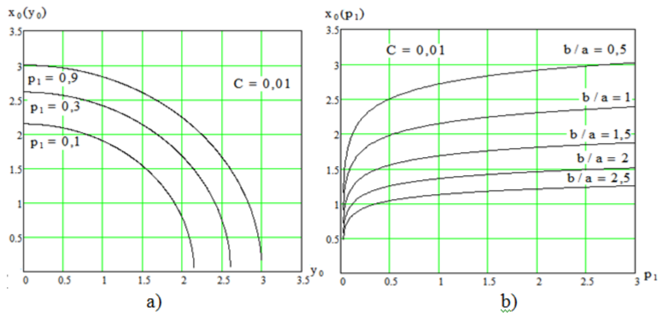

Figure 2: Dependence curves: a) optimal threshold of detector from optimal threshold of recognizer; b) optimal threshold of detector from a priori probability of networks normal status.

Thereby, when its fluctuation of navigation area, we must establish the fact of fluctuating and, after that, identify state of transionospheric communication channels for develop necessary control procedure. This will allow to hold signal/noise of GPS/GLONASS in required values.

Dependences in Fig. 2a, built according to the formula (16) are circumferences of the $\sqrt{2lnp_1/C}$ radius. With the increase of a priori probability of normal connection status p_1, the radius is increased subject to the logarithmic law.

6 Conclusions

The problem of optimizing classification thresholds of the first and second kind of errors of trans-ionospheric communication channels control and monitoring system, described by the Rayleigh distribution law of random variables is solved in this paper. The authors have obtained the common solution of problem the optimal classification thresholds for the anomalous situation detector and recognizer. In future work, it is advisable to conduct an investigation on the dependencies of the communication channel states for the distribution law of random variables described by Nakagami distribution functions.

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